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Superconformal and super B.R.S. invariance of the N=1 supersymmetric W. Z. W. model based on Lie superalgebra*

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Abstract

We study the superconformal and super B.R.S. invariance of supersymmetric Wess Zumino Witten model based on Lie superalgebra. The computation of the critical super dimension of this model is done using the Fujikawa regularisation. finally, we recover the well known result which fixes the relative coupling constant $\alpha^2 = 1$ in a rigorous way.

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1 Introduction

The Wess Zumino Witten model [1] in two dimensions plays a central role in many current investigations. A tremendous amount of literature has been devoted to this model [2]. In this paper, we study the superconformal and super B.R.S. invariance of the supersymmetric extension of this model based on Lie superalgebra using the Fujikawa regularization [3]. The regularization allows us to evaluate the anomalous superconformal and super B.R.S. currents extending the results of Ref [4] to N=1 supersymmetric case. The expression of the super Jacobian exhibits forms of anomalies which are in agreement with several results based essentially on the computation of the supersymmetric function at one loop level [5]. We recover the well known result which fixes the relative coupling constant $\alpha^2 = 1$ in a rigorous way.

This paper is organized as follows: In Sec 2, we present the Wess Zumino Witten action. In Sec 3, we study its invariance under the superconformal transformations and their corresponding super B.R.S transformations. In Sec 4, we compute the super Jacobian of the holomorphic super B.R.S transformations of the path integral measure and we derive the super B.R.S identity using the Fujikawa regularization.

2 The Wess Zumino Witten action

Let us first consider the action describing this model [5]. In the covariant superconformal gauge [7], this action is given by:

$$A_U = \int_{\Sigma_2} \frac{d^2 z d^2 \theta}{8\pi\alpha'} \frac{R^2}{2} \text{Tr} (\overline{D} U D U^{-1}) + \frac{k}{N_G} \int_{\Sigma_3} dt d^2 \xi d^2 \theta \text{Tr} U^{-1} \dot{U} (U^{-1} D^\alpha U) (-i\gamma_5)^\alpha_\beta (U^{-1} D_\beta U), \quad (1)$$

where U is an element of a generic compact Lie supergroup G associated to a Lie superalgebra g, D and \overline{D} are complex super derivatives, Σ_3 is a three super Riemann surface and Σ_2 its boundary ($\partial\Sigma_3 = \Sigma_2$) and N_G is a normalization factor. α' is the Regge slope which is related to the string tension by: $T = \frac{1}{2\pi\alpha'}$ and R is the radius of the compact group supermanifold. It measures the size of the supermanifold. Low energy states are associated with the propagation of the particles on the Minkowski space time M^4 . Any propagation on G will require energies $\sim hc/R$. Such states would be unexcited by probes of energy $\ll hc/R$ and the internal space would therefore be invisible. Then if one assumes the motion space to be a direct product of Minkowski space with an internal group space G, we may tentatively choose the dimension of Minkowski space to

$d = 4$ by a suitable choice of the internal group G . If this is done, the model turn out to be a superstring field theory in four dimensional space-time. This little digression has a great significance in the mechanism of compactification of string theory [7] and Kaluza-Klein theories [8]. A proper job would require us to go into details that are not crucial to the present discussion.

Although the action specified in Eq (1) is multivalued, the factor $\exp(-A_U)$ is unique, provided the coupling constant k in front of the Wess Zumino term is an integer. The action (1) can be seen as the action of a closed type II supersymmetric string (two-dimensional supergravity) or supersymmetric non linear defined on a compact group supermanifold built on Lie superalgebra in the presence of the Wess Zumino term. This latter term was originally introduced by Witten [1] to restore the conformal invariance at the quantum level. In the context of non-abelian bosonization, Witten has shown that if the relation: $\frac{\alpha'}{R^2} = \frac{N_G}{24|k|}$, i.e. $\alpha^2 = 1$ is satisfied, conformal invariance and hence reparametrization invariance is restored. This relation means that the radius of the compactified dimensions gets quantized in units of the string tension. Furthermore, when k approaches infinity one recovers the flat space limit. The Wess Zumino term is topological and can be also interpreted as a torsion which parallelizes the curvative at the point which the β -function vanishes [5].

3 Gauge fixing

The covariant superconformal gauge choice yields to the action of the Fadeev-Popov ghost superfield system (B, C) of superconformal spin $(3/2, -1)$

$$A_{gh} = \int \frac{d^2 z d^2 \theta}{4\pi\alpha'} (B\overline{D}C + \overline{B}D\overline{C}). \quad (2)$$

Under the superconformal transformations parametrized by infinitesimal super vector fields V and \overline{V} satisfying the conditions:

$$\overline{\partial}V = \overline{D}V = D\overline{V} = 0 \quad (3)$$

the superfields variables of matter, the ghost and the superconformal factor transform as [9]:

$$\delta U = V\partial U + \frac{1}{2}(DV)DU + \overline{V}\partial U + \frac{1}{2}(\overline{D}V)\overline{D}U, \quad (4)$$

$$\delta C = V\partial C + \frac{1}{2}(DV)DC - (\partial V)C + \overline{V}\partial C + \frac{1}{2}(\overline{DV})\overline{DC}, \quad (5)$$

$$\delta \overline{C} = V\partial \overline{C} + \frac{1}{2}(DV)D\overline{C} - (\partial V)\overline{C} + \overline{V}\partial \overline{C} + \frac{1}{2}(\overline{DV})\overline{DC}, \quad (6)$$

$$\delta B = V\partial B + \frac{1}{2}(DV)DB + \frac{3}{2}(\partial V)B + \overline{V}\partial B + \frac{1}{2}(\overline{DV})\overline{DB}, \quad (7)$$

$$\delta \overline{B} = V\partial \overline{B} + \frac{1}{2}(DV)D\overline{B} + \frac{3}{2}(\overline{\partial V})\overline{B} + \overline{V}\partial \overline{B} + \frac{1}{2}(\overline{DV})\overline{DB}, \quad (8)$$

$$\delta \Psi = V\partial \Psi + \frac{1}{2}(DV)D\Psi - \frac{1}{4}(\partial V) + \overline{V}\partial \Psi + \frac{1}{2}(\overline{DV})\overline{D\Psi} - \frac{1}{4}\partial \overline{V}. \quad (9)$$

Under these transformations, the action changes as:

$$\delta A_{U+gh} = \int \frac{d^2 z d^2 \theta}{4\pi\alpha'} [V\overline{D}(V\overline{D}(T^U + T^{gh})_{z\theta}) + h.c.], \quad (10)$$

where $T_{z\theta}^U + T_{z\theta}^{gh}$ is the super stress-energy tensor given by:

$$T_{z\theta}^U = -\frac{R^2}{2}Tr(\partial U D U^{-1}), \quad (11)$$

$$T_{z\theta}^{gh} = -C\partial B + \frac{1}{2}(DC)DB - \frac{3}{2}(\partial C)B. \quad (12)$$

These currents are separately conserved:

$$\overline{D}T_{z\theta}^U = \overline{D}T_{z\theta}^{gh} = D\overline{T}_{z\theta}^U = D\overline{T}_{z\theta}^{gh} = 0. \quad (13)$$

The action A_{U+gh} is also invariant under the super-BRS transformations obtained by the replacement, in the superconformal transformations of U and Ψ , of V and \overline{V} by respectively ϵC and $\overline{\epsilon}\overline{C}$:

$$\delta U = \epsilon C \partial U - \frac{1}{2} (DC) DU + \bar{\epsilon} \bar{C} \bar{\partial} U - \frac{1}{2} \bar{\epsilon} (\bar{DC}) \bar{D} U, \quad (14)$$

$$\delta \Psi = \epsilon C \partial \Psi - \frac{1}{2} \epsilon (DC) D \Psi - \frac{1}{4} \epsilon \partial C + \bar{\epsilon} \bar{C} \bar{\partial} \Psi - \frac{1}{2} \bar{\epsilon} (\bar{DC}) \bar{D} \Psi - \frac{1}{4} \bar{\epsilon} \bar{\partial} C, \quad (15)$$

where ϵ is a Grassmann parameter. For the ghost superfield system, the super-BRS transformations are defined as:

$$\delta C = \epsilon C \partial C - \frac{1}{4} \epsilon (DC) (DC) + \bar{\epsilon} \bar{C} \bar{\partial} C - \frac{1}{4} \bar{\epsilon} (\bar{DC}) (\bar{DC}), \quad (16)$$

$$\delta \bar{C} = \epsilon C \partial \bar{C} - \frac{1}{4} \epsilon (DC) (D \bar{C}) + \bar{\epsilon} \bar{C} \bar{\partial} \bar{C} - \frac{1}{4} \bar{\epsilon} (\bar{DC}) (\bar{DC}), \quad (17)$$

$$\delta B = \epsilon (T^U + T^{gh}) + \bar{\epsilon} \bar{C} \bar{\partial} B - \frac{1}{2} \bar{\epsilon} (\bar{DC}) \bar{D} B, \quad (18)$$

$$\delta \bar{B} = \bar{\epsilon} (\bar{T}^U + \bar{T}^{gh}) + \epsilon C \partial \bar{B} - \frac{1}{2} \epsilon (DC) D \bar{B}. \quad (19)$$

All these invariances are verified in the holomorphic sector where we consider the variations, with $\bar{V} = \bar{\epsilon} = 0$, of the spinning string variable and its ghost only with the superconformal factor Ψ and the anti-ghost superfield system kept fixed.

Under the localised holomorphic super B.R.S transformations which we will consider in the next section,

$$\delta U = \epsilon(x) C \partial U - \frac{1}{2} \epsilon(x) (DC) (DU), \quad (20)$$

$$\delta C = \epsilon(x) C \partial C - \frac{1}{4} \epsilon(x) (DC) (DC), \quad (21)$$

$$\delta B = \epsilon(x) (T^U + T^{gh}), \quad (22)$$

$$\delta\overline{B} = \delta\overline{C} = \delta\Psi = 0, \quad (23)$$

the action changes as

$$\begin{aligned} \delta A_{U+gh} &= \int \frac{d^2z d^2\theta}{4\pi\alpha'} \left[(\overline{D}\epsilon) J_\theta + \frac{1}{4} (\overline{D}\epsilon) D (BCDC) \right] + \int \frac{d^2z d^2\theta}{4\pi\alpha'} (D\epsilon) \\ &\times \left[-C [\overline{D}DU DU^{-1}] - D (CB\overline{D}C) + \frac{1}{2} BDCD\overline{C} \right], \end{aligned} \quad (24)$$

with the canonical super B.R.S current

$$J_\theta = C \left(T_{z\theta}^U + \frac{1}{2} T_{z\theta}^{gh} \right) = \frac{-R^2}{2} C \partial U D U^{-1} - C \left[\frac{3}{4} BDC - \frac{1}{4} (DB) DC \right]. \quad (25)$$

This current is conserved when we use the classical equations of motion for the superfields U, B and C.

Before we close this section, we make one remark concerning the superfield U. In the quantum theory, the superfield fluctuates around the classical solution, and to analyse these fluctuations, we set:

$$U(z, \bar{z}, \theta, \bar{\theta}) = U_{cl}(z, \bar{z}, \theta, \bar{\theta}) U_q(z, \bar{z}, \theta, \bar{\theta}) \quad (26)$$

so that $U_q(z, \bar{z}, \theta, \bar{\theta})$ fluctuates around $U_q(z, \bar{z}, \theta, \bar{\theta}) = 1$. We treat the amplitude of the space-independent fluctuations as a collective variable, setting $U_q(z, \bar{z}, \theta, \bar{\theta}) = u_0 \cdot \exp(X(z, \bar{z}, \theta, \bar{\theta}) + Y(z, \bar{z}, \theta, \bar{\theta}))$. Here u_0 is the collective variable associated with zero modes, while $X + Y$ describes the fluctuations in the nonzero modes. The superfield U is then given by :

$$U = U_{cl} \cdot u_0 \cdot \exp(X + Y) = U_0 \cdot \exp(X + Y) = U_0 \cdot \exp(X^A T_A + Y^a S_a), \quad (27)$$

where the bosonic generators,

$$T_A, \quad A = 1, \dots, D_G \quad (28)$$

and the fermionic generators

$$S_a, \quad a = 1, \dots, d_G \quad (29)$$

span the superalgebra \mathfrak{g} associated to a generic Lie supergroup G . The Lie product

$$[T_A, T_B] = f_{AB}^C T_C, \quad (30)$$

$$[S_a, S_b]_+ = g_{ab}^A T_A, \quad (31)$$

$$[S_a, T_A] = -[T_A, S_a] = h_{aA}^b S_b \quad (32)$$

fulfills the Jacobi identities

$$f_{AB}^D f_{CD}^E + f_{BC}^D f_{AD}^E + f_{CA}^D f_{BD}^E = 0, \quad (33)$$

$$h_{aC}^b f_{AB}^C + h_{aB}^c h_{cA}^b - h_{aA}^c h_{cB}^b = 0, \quad (34)$$

$$h_{bA}^c g_{ac}^B + h_{aA}^c g_{bc}^B + g_{ab}^C f_{AC}^B = 0, \quad (35)$$

$$g_{bc}^A h_{aA}^d + g_{ca}^A h_{bA}^d + g_{ab}^A h_{cA}^d = 0. \quad (36)$$

The generators T_A and S_a are normalized as:

$$Tr(T_A T_B) = 2\delta_{AB}, \quad (37)$$

$$Tr(S_a S_b) = 2\varepsilon_{ab}, \quad (38)$$

$$Tr (T_A S_a) = Tr (S_a T_A) = 0. \quad (39)$$

Notice that $X^A T_A = T_A X^A$ since they are bosons and $Y^a S_a = -S_a Y^a$ since they are fermions. The superfields and transform under the superconformal transformations as:

$$\delta X^A = V \partial X^A + \frac{1}{2} (DV) DX^A + \overline{V} \overline{\partial} X^A + \frac{1}{2} (\overline{DV}) \overline{D} X^A, \quad (40)$$

$$\delta Y^a = V \partial Y^a + \frac{1}{2} (DV) DY^a + \overline{V} \overline{\partial} Y^a + \frac{1}{2} (\overline{DV}) \overline{D} Y^a \quad (41)$$

and transform under the super B.R.S transformations as:

$$\delta X^A = \epsilon C \partial X^A - \frac{1}{2} \epsilon (DC) DX^A + \overline{\epsilon} \overline{C} \overline{\partial} X^A - \frac{1}{2} \overline{\epsilon} (\overline{DC}) \overline{D} X^A, \quad (42)$$

$$\delta Y^a = \epsilon C \partial Y^a - \frac{1}{2} \epsilon (DC) DY^a + \overline{\epsilon} \overline{C} \overline{\partial} Y^a - \frac{1}{2} \overline{\epsilon} (\overline{DC}) \overline{D} Y^a. \quad (43)$$

In terms of the X^A and Y^a , the action becomes:

$$\begin{aligned} A_U (U_0, X^A, Y^a) &= A(U_0) + \int \frac{d^2 z d^2 \theta}{4\pi\alpha'} R^2 [-\overline{D} X^A DX^B \delta_{AB} + \overline{D} Y^a DY^b \epsilon_{ab}] \\ &+ \int \frac{d^2 z d^2 \theta}{8\pi\alpha'} R^2 [\frac{f_{AB}^C}{2} [Tr (U_0^{-1} \overline{D} U_0 T_C) X^A DX^B (1 - \alpha) \\ &+ Tr (U_0^{-1} D U_0 T_C) X^A \overline{D} X^B (1 + \alpha)] \\ &- \frac{g_{ab}^A}{2} [Tr (U_0^{-1} \overline{D} U_0 T_A) Y^a DY^b (1 - \alpha) \\ &+ Tr (U_0^{-1} D U_0 T_A) Y^a \overline{D} Y^b (1 + \alpha)] \\ &+ \frac{h_{aA}^b}{2} [Tr (U_0^{-1} \overline{D} U_0 S_b) X^A DY^a (1 - \alpha) \\ &+ Tr (U_0^{-1} D U_0 S_b) X^A \overline{D} Y^a (1 + \alpha)] \\ &- \frac{h_{aA}^b}{2} [Tr (U_0^{-1} \overline{D} U_0 S_b) Y^a DX^A (1 - \alpha) \\ &+ Tr (U_0^{-1} D U_0 S_b) Y^a \overline{D} X^A (1 + \alpha)], \end{aligned} \quad (44)$$

where

$$\begin{aligned}
A(U_0) &= \int_{\Sigma_2} \frac{d^2 z d^2 \theta}{8\pi\alpha'} \frac{R^2}{2} (\overline{D}U_0 D U_0^{-1}) \\
&\quad + \frac{k}{N_G} \int_{\Sigma_3} dt d^2 \xi d^2 \theta \text{Tr} U_0^{-1} \dot{U}_0 (U_0^{-1} D^\alpha U_0) (-i\gamma_5)_\alpha^\beta (U_0^{-1} D_\beta U_0),
\end{aligned} \tag{45}$$

which gives:

$$A_U(U_0, X^A, Y^a) = \int \frac{d^2 z d^2 \theta}{4\pi\alpha'} L(U_0, X^A, Y^a) \tag{46}$$

up to terms cubic or higher order in X^A and Y^a . We restricted our selves to the quadratic order and used the classical equations of motion to eliminate the first-order terms.

4 Generating functional and super B.R.S identity

Following, Polyakov [10] the partition function of the model at hand is given in the superconformal gauge by:

$$Z = \int D_\mu e^{-A(\tilde{X}^A, \tilde{Y}^a, \tilde{B}, \tilde{C})}, \tag{47}$$

where D_μ is the reparametrization invariant measure and we have defined the new superfields as:

$$\tilde{X}^A = e^{-\Psi} X^A, \tag{48}$$

$$\tilde{Y}^a = e^{-\Psi} Y^a, \tag{49}$$

$$\tilde{C} = e^{-3\Psi} C, \tag{50}$$

$$\widetilde{\overline{C}} = e^{-3\Psi} \overline{C}, \quad (51)$$

$$\widetilde{B} = e^{2\Psi} B, \quad (52)$$

$$\widetilde{\overline{B}} = e^{2\Psi} \overline{B}. \quad (53)$$

This method is completely analogous to that followed by Fujikawa [3]. One may start with variables with suitable weight factors so the resulting path integral measure is formally invariant under diffeomorphisms.

The way to derive super B.R.S identity, which is insensitive to the definition of the path integral variable is to start with the localized variations:

$$\delta \widetilde{X}^A = \epsilon(x) \left[C \partial \widetilde{X}^A - \frac{1}{2} (DC) D \widetilde{X}^A + C (\partial \Psi) \widetilde{X}^A - \frac{1}{2} (DC) (D \Psi) \widetilde{X}^A \right], \quad (54)$$

$$\delta \widetilde{Y}^a = \epsilon(x) \left[C \partial \widetilde{Y}^a - \frac{1}{2} (DC) D \widetilde{Y}^a + C (\partial \Psi) \widetilde{Y}^a - \frac{1}{2} (DC) (D \Psi) \widetilde{Y}^a \right], \quad (55)$$

$$\begin{aligned} \delta \widetilde{B} &= \epsilon(x) \left[C \partial \widetilde{B} - \frac{1}{2} (DC) D \widetilde{B} - 2C (\partial \Psi) \widetilde{B} + (DC) (D \Psi) \widetilde{B} \right. \\ &\quad \left. + \frac{3}{2} (\partial C) \widetilde{B} - e^{2\Psi} T_{z\theta}^{X+Y} \right], \end{aligned} \quad (56)$$

$$\delta \widetilde{C} = \epsilon(x) \left[C \partial \widetilde{C} - \frac{3}{4} (DC) (D \Psi) \widetilde{C} - \frac{1}{4} (DC) D \widetilde{C} \right], \quad (57)$$

$$\delta \widetilde{\overline{B}} = \delta \widetilde{\overline{C}} = \delta \Psi = 0 \quad (58)$$

To evaluate the super Jacobian factor resulting from these transformations we use the differential of

$$\begin{aligned}
d\delta\tilde{C} &= \epsilon(x) \left[C\partial\tilde{C} - \frac{1}{2}(DC)D\left(d\tilde{C}\right) + 3C(\partial\Psi)d\tilde{C} \right. \\
&\quad \left. - \frac{3}{2}(DC)(D\Psi)d\tilde{C} - (\partial C)D\tilde{C} \right]
\end{aligned} \tag{59}$$

to obtain:

$$\begin{aligned}
\ln J &= Str \epsilon(x) \left[C\partial - \frac{1}{2}(DC)D + C(\partial\Psi) - \frac{1}{2}(DC)(D\Psi) \right]_{X^A} \\
&\quad + Str \epsilon(x) \left[C\partial - \frac{1}{2}(DC)D + C(\partial\Psi) - \frac{1}{2}(DC)(D\Psi) \right]_{Y^a} \\
&\quad + Str \epsilon(x) \left[C\partial - \frac{1}{2}(DC)D + 2C(\partial\Psi) + (DC)(D\Psi) + \frac{3}{2}(\partial C) \right]_B \\
&\quad - Str \epsilon(x) \left[C\partial - \frac{1}{2}(DC)D + 3C(\partial\Psi) - \frac{3}{2}(DC)(D\Psi) - (\partial C) \right]_C
\end{aligned} \tag{60}$$

The super-Jacobian factor of the ghost superfields, which are free, have been evaluated in Ref. 3. For the Jacobian of the superfields X^A and Y^a we must regularize the ill-defined supertrace of the unity, $Str(1) = \sum_{n=1}^{\infty} \sum_{A=1}^{D_G} \tilde{\phi}_n^A \tilde{\phi}_n^A$ and $Str(1) = \sum_{n=1}^{\infty} \sum_{a=1}^{d_G} \tilde{\phi}_n^a \tilde{\phi}_n^a$. $\tilde{\phi}_n^A$ and $\tilde{\phi}_n^a$ belongs to a closed set of eigenfunctions of the Hermitian regulators:

$$H_{AB}\tilde{\phi}_n^B = \lambda_n^2 \cdot \delta_{AB} \cdot \tilde{\phi}_n^B, \tag{61}$$

$$H_{ab}\tilde{\phi}_n^b = \lambda_n^2 \cdot \delta_{ab} \cdot \tilde{\phi}_n^b, \tag{62}$$

where

$$H_{AB}\tilde{\phi}_n^B = \frac{\delta L}{\delta \tilde{\phi}_n^A}, \tag{63}$$

$$H_{ab}\tilde{\phi}_n^b = \frac{\delta L}{\delta \tilde{\phi}_n^a}, \tag{64}$$

L is defined in (46). The regulators H_{AB} and H_{ab} are given by:

$$\begin{aligned}
H_{AB} = & e^\psi \overline{D} D \delta_{AB} e^\psi - \frac{f_{ABC}}{4} [Tr (U_0^{-1} \overline{D} U_0 T^C) e^\psi D e^\psi (1 - \alpha) \\
& + Tr (U_0^{-1} D U_0 T^C) e^\psi \overline{D} e^\psi (1 + \alpha)],
\end{aligned} \tag{65}$$

$$\begin{aligned}
H_{ab} = & e^\psi \overline{D} D \delta_{ab} e^\psi - \frac{g_{abc}}{4} [Tr (U_0^{-1} \overline{D} U_0 T^C) e^\psi D e^\psi (1 - \alpha) \\
& + Tr (U_0^{-1} D U_0 T^C) e^\psi \overline{D} e^\psi (1 + \alpha)].
\end{aligned} \tag{66}$$

There is no conceptual difficulty in the formulation of the regularization since the regulators do not depend on the quantum fluctuations X^A and Y^a . This is a result of the expansion in (44) which gives us a quadratic action.

The evaluation of the ill defined supertrace in (60) is standard. One may use the basis of the super plane waves $e^{(ik\bar{z} + i\bar{k}z) - \chi\bar{\theta} - \overline{\chi}\theta}$, introduces a cut-off M , rescales the variables $k \rightarrow Mk$, $\bar{k} \rightarrow M\bar{k}$, $\chi \rightarrow M\chi$, $\overline{\chi} \rightarrow M\overline{\chi}$ and the fact that the integration measure is invariant under this simultaneous rescaling of bosonic and fermionic variables [3]. This straightforward but tedious calculation leads to:

$$\begin{aligned}
\ln J = & \frac{1}{2\pi} \int d^2 z d^2 \theta \epsilon(x) [5 - (D_G - d_G) [C (\partial D \overline{D} \psi) + 2C (\partial \psi) (D \overline{D} \psi)] \\
& + 10C (D \psi) (\partial \overline{D} \psi) + \frac{1}{2} (D_G - d_G) (DC) (\partial \overline{D} \psi) \\
& + (D_G - d_G) (DC) (D \psi) (D \overline{D} \psi)] \\
& + \frac{C_V \cdot D_G}{16\pi} (1 - \alpha^2) \int d^2 z d^2 \theta \epsilon(x) [C [\partial Tr (\overline{D} U_0 D U_0^{-1}) - \partial \psi Tr (\overline{D} U_0 D U_0^{-1})] \\
& - \frac{1}{2} (DC) [D Tr (\overline{D} U_0 D U_0^{-1}) - D \psi Tr (\overline{D} U_0 D U_0^{-1})]],
\end{aligned} \tag{67}$$

where the constant C_V is related to the dual Coxeter number of the super-algebra and is given by:

$$f_{AB}^C f_{AB}^D \delta_{CD} + g_{ab}^A g_{ab}^B \delta_{AB} = C_V \cdot D_G. \tag{68}$$

As usual, the Ward-Takahachi identity is obtained by equating the variation of the action with the variation of the measure. If we dodge the intricate

details of the well known identities [3] which are pertinent to our cumbersome calculation, we obtain, finally, the basic super B.R.S identity:

$$\begin{aligned}
\overline{D}J_\theta &= \overline{D}\langle (D_G - d_G - 10) [C (\partial\psi) D\psi + \frac{1}{2}C (\partial D\psi)] - 4D[C\partial\psi - \frac{1}{2}(DC) D\psi] \rangle \\
&+ \frac{C_V \cdot D_G}{8} (1 - \alpha^2) [\langle C[\partial Tr(\overline{D}U_0 D U_0^{-1}) - \partial\psi Tr(\overline{D}U_0 D U_0^{-1})] \\
&- \frac{1}{2}(DC) [DTr(\overline{D}U_0 D U_0^{-1}) - D\psi Tr(\overline{D}U_0 U_0^{-1})] \rangle]. \tag{69}
\end{aligned}$$

This super B.R.S identity exhibits three kinds of anomalies:

(a) It is proportional to $(D_G - d_G - 10)$ which determines the well-known critical dimension for the consistency of the quantum model based on a super-Lie algebra in a flat space. The dimension $D_G - d_G$ is the same as the dimension found by Fujikawa et al. [3] and the value $-d_G$ is in agreement with the result found by Henningson [11] who studied this model using the covariant quantization method.

(b) It is a total divergence. It is related to the ghost number anomaly, and exist also in the free case.

© It can be eliminated only if $\alpha^2 = 1$ for supergroups with $C_V \neq 0$. The addition of a Wess Zumino term to a non linear sigma model defined on a supergroup manifold leads to a free theory for a certain value of the relative coupling constant of the two terms in the action ($\alpha^2 = 1$). In that case the full action is super conformally invariant.

5 Conclusion

The principles of conformal and B.R.S invariance have deep consequences for string theory and in quantization of gauge theories. In the present paper, we have presented the detailed B.R.S analyses of a closed type II supersymmetric string (two-dimensional supergravity) or supersymmetric non linear sigma-model defined on a group supermanifold built on Lie superalgebra in the presence of the Wess Zumino term. By a direct evaluation of the regularization of the super Jacobian corresponding to localised holomorphic super-BRS transformations of the superstring and its ghosts, we found that this model is quantum mechanically consistent only in the case where the relative coupling constant satisfies the well-known relation $\alpha^2 = 1$ for supergroups with $C_V \neq 0$. We are not surprised to find that this value is the same as the infrared fixed point of the bosonic sector of the theory [4], the critical point where the chiral Bose theory with Wess-Zumino term is equivalent to free Fermi theory [1]. Finally, let us

emphasise once more that our results are in perfect agreement with several results based essentially on the computation of the supersymmetric beta function at one-loop level [5].

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